GEOMETRY IN INDIAN MATHEMATICS

A DROP FROM THE OCEAN OF GEOMETRY

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IMPORTANCE

- *Vedāṅga Jyotiṣa* (v.4) records:
- **यथा शिखा मयूराणां नागानां मणयो यथा**
  तद्वद्वेदाङ्गशास्त्राणां गणितं मूर्धनि स्थितम् ॥

- *Gaṇita-sāra-saṅgraha* (*GSS*.I.16) that all the moving and non-moving beings in all the three worlds cannot exist without Mathematics :
- **बहुभिर्विप्रलापैः किं त्रैलोक्ये सचराचरे ।**
  यत्किंचिद्वस्तु तत्सर्वं गणितेन विना नहि ॥

- Geometry is termed as *Kṣetra vyavahāra* in IM. This includes
- Constructions,
- circumference and area of plane figures,
- volume of solids,
- Shadows,
- geometrical results.

- Topics related to geometry are found in *Śulba sūtras* (earlier than 800 BCE), *Āryabhaṭīya* (499 CE), *BrāhmaSphuṭa Siddhānta* (628 CE), *Gaṇita-sāra-samgraha* (850 CE), *Līlāvatī* (1050 CE), *GaṇitaKaumudī* (1346 CE) and so on.
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Plane figures

- क्षेत्रं नाम समभूमिः । तदतिदेशत्वेन यत्किंचित्त्रिकोणप्रदेशादिकं तत् । तस्य व्यवहारः कर्णलम्बफलादिभिरियता निर्णयः ।
- *Kṣetram* means a flat surface. Applying by analogy (to geometrical figures), whatever triangular field etc. that (is considered). Triangle etc. are denoted as plane figures. Its investigation is determining the measure of the hypotenuse (diagonal), altitude, area etc.

Classification of Kṣetra

Kṣetra

Triangle

(tryasra)

Jātya (1) (Right)

Tribhuja (others)

- Antarlamba (acute) (2)
- Bahirlamba (obtuse) (3)

Quadrilateral

(caturasra)

Samakarṇa (with equal diagonals)

4. **samacaturbhuja** (square)
5. **viṣamacaturbhuja**
6. **āyata**(**rectangle**)
7. **āyata-samalamba**

Viṣamakarṇa (with unequal diagonals)

8. samacaturbhuja **(rhombus)**
9. samatribhuja **(3 sides equal)**
10. samadvi-dvibhuja**(2 pairs of sides equal)**
11. samadvibhuja **(2 sides equal)**
12. viṣama-caturbhuja **(unequal sides)**
13. sama lamba **(trapezium)**

Circle&Arc

(vṛtta) (14)

GEOMETRICAL CONSTRUCTIONS.

Śulba sūtras

- *Śulba sūtras* (earlier than 800BC) are the oldest available geometrical treatises.
- This has been developed for construction and transformation of vedic altars of various shapes.
- Baudhāyana, Āpastamba, Kātyāyana, Mānava and many others are the authors of the *Śulba sūtras.*
- Most of the constructions that we construct today are given in the *Śulba sūtras.*
- Construction of fire altars (vedi) in the form of triangles, quadrilaterals, circles etc., transformations of square equal in area to a rhombus, circle etc. and vice versa; one figure equal in area to the other.

Transforming square into a rectangle

- *Baudhyana Śulbasūtra* (Ch 1.v.52) gives rule for transforming square into a rectangle of equal area:
- समचतुरश्रं दीर्घचतुरस्त्रं चिकीर्षन् अस्तदक्ष्णयापच्छिद्य भागं द्वेधा विभज्य पार्श्वयोरुपदध्याद्यथापयोग्यम् ||
- A square intended to be transformed into a rectangle is cut off by its diagonal. One portion is divided into two equal parts which are placed on the two sides of the other potion so as to fit them exactly.

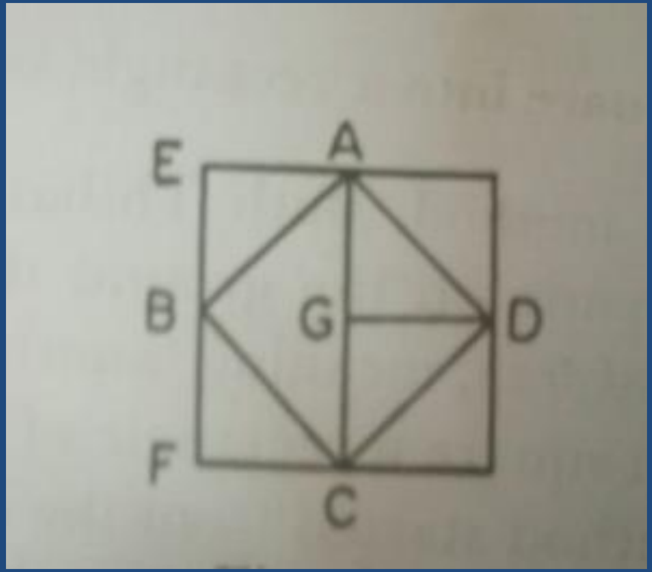
The square is transformed into a rectangle such that the diagonal of the square equals the longer side of the rectangle.

The square ABCD is divided by its diagonal AC. The portion ADC is again divided into two equal halves by GD and each is transferred to occupy the position AEB and BFC. The AEFC is the required rectangle . For,

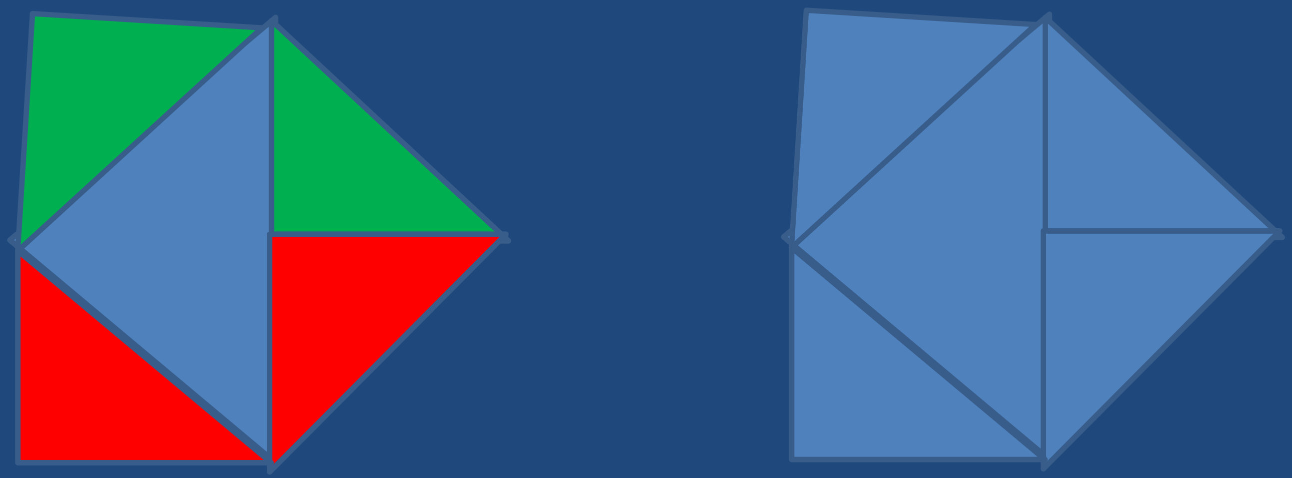
Sq. ABCD = tr.ABC + tr.AGD + tr.GCD

= tr.ABC + tr.AEB + tr.BFC

= rect.AEFC

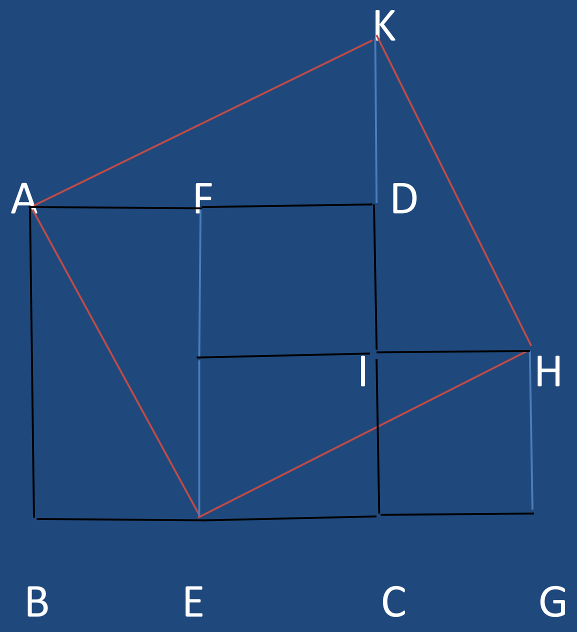


Area of rectangle = area of square

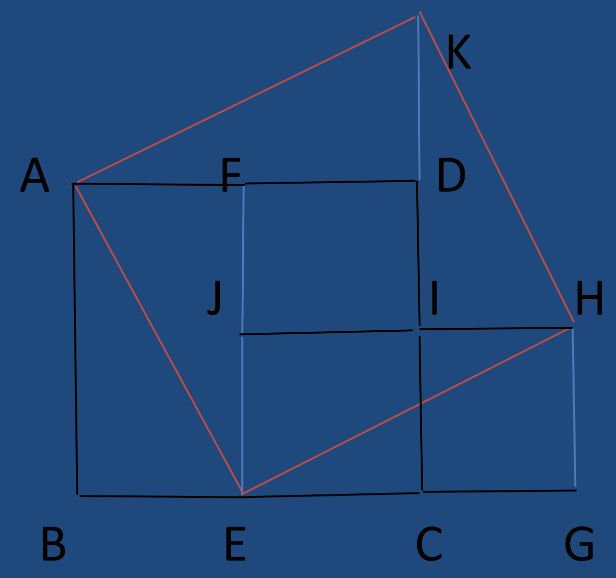


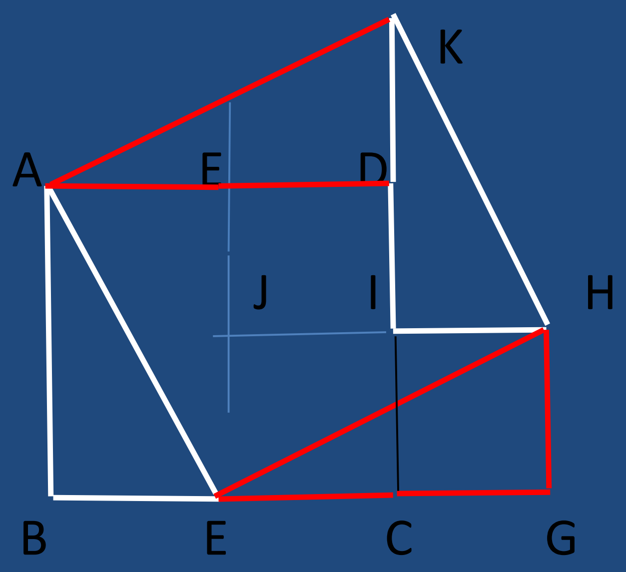
Constructing a square equal to Sum of Two Squares

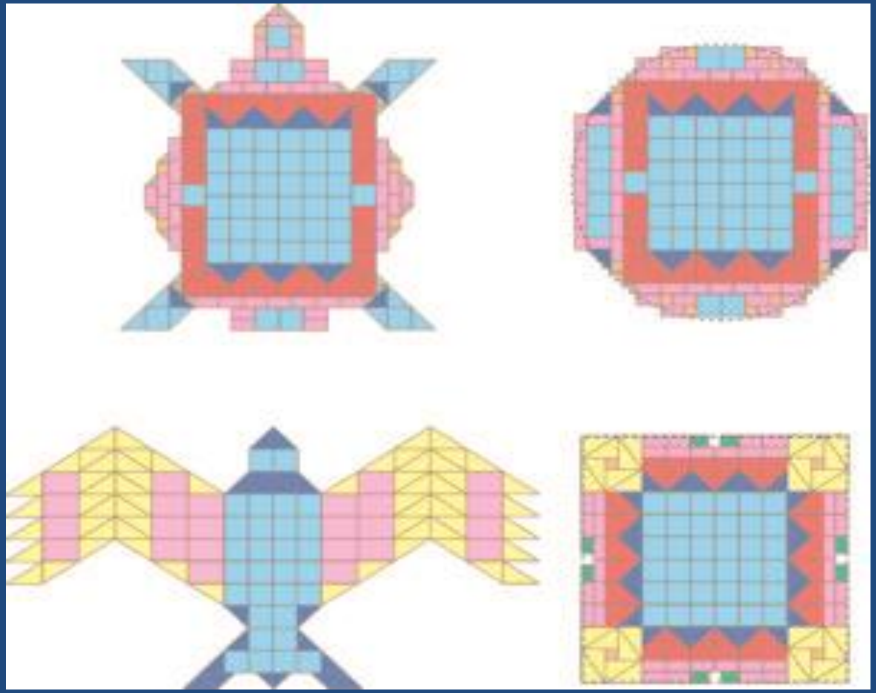
- नानाचतुरश्रे समस्यन् कनीयसः करण्या वर्षीयसौ वृध्रमुल्लिखेत् । वृध्रस्य अक्ष्णया रज्जुः समस्यतोः पार्श्वमानीं भवति। (BSS 1.50)
-
- ‘To combine two different squares,
- mark a rectangle in the bigger square
- with a side equal to side of smaller
- square. The diagonal of this is the side
- of the sum of two given squares,’
- AE^2 = ABCD + CGHI
  $= AB^2 + CG^2$
  $= AB^2 + BE^2$
-



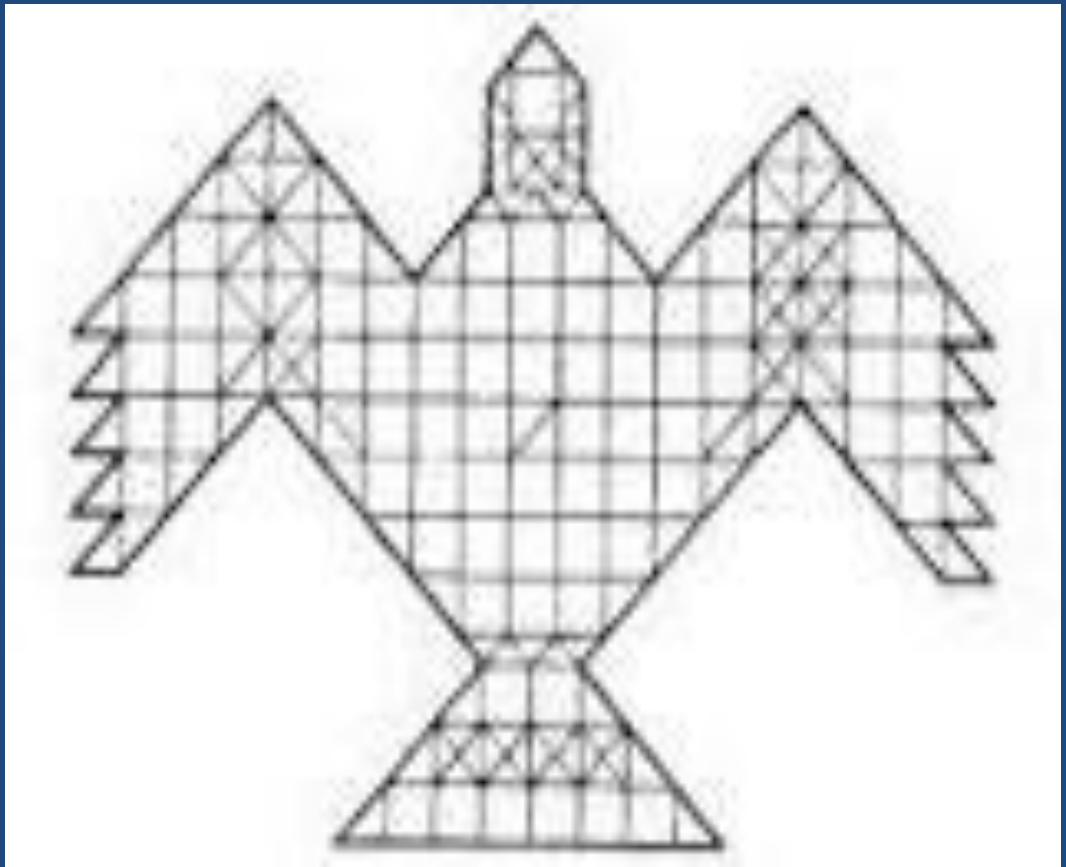
Area of sum of two squares = one square
Area of(ABCD +ICGH)= Ar(ADIHE +ABE+EGH) =Ar(ADIHE+KIH+ADK)
Area of AEHK = AE^2

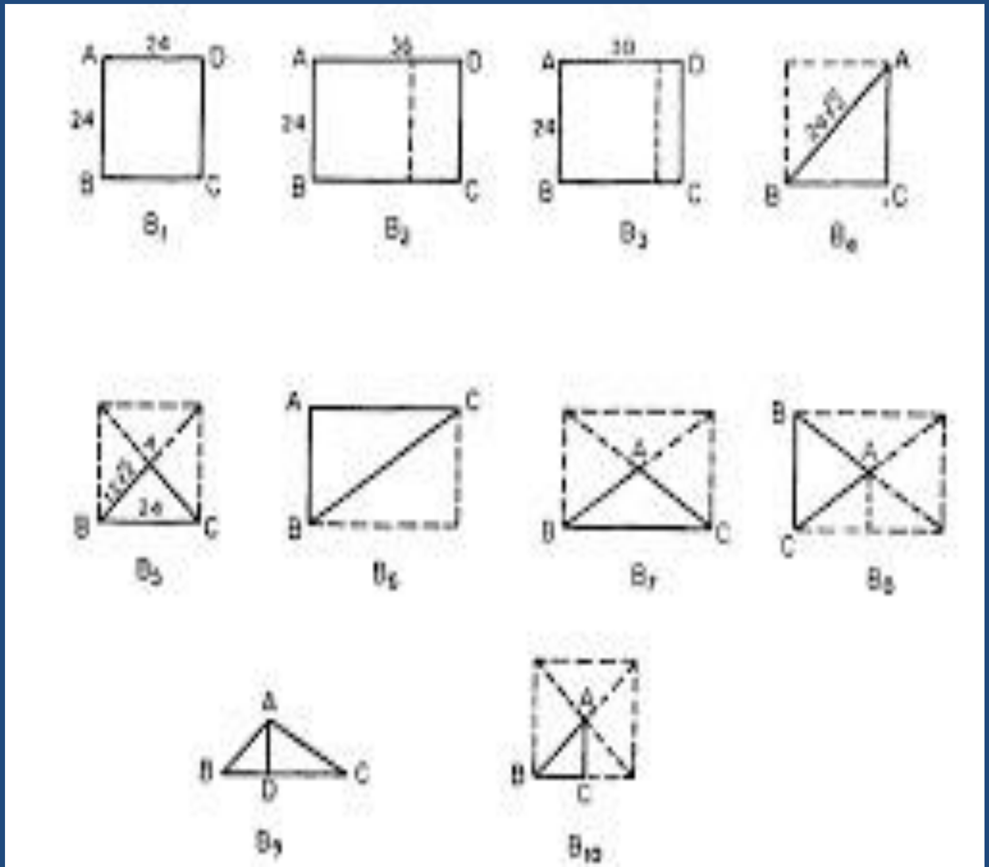




Measurements of the bricks used in Śyena citi, as given in *Śulbasūtras*





Excavation of *Śyenaciti*

- Recently *Śyenaciti* shaped fire-altar has been excavated at Purola, Uttarkhand, which is dated around 1st century BCE, the picture of which is given below

Drawing a circle equal in area to a square

- *Mānava Śulbasūtra* (11.15) explains:
- चतुरस्रं नवधा कुर्याद् धनुः कोट्यास्त्रिधात्रिधा ।
  उत्सेधात्पामममंलुम्पेत्पुरीषेणेह तावत् समम् ॥
- "Divide the square in to nine parts by drawing three (parallel) lines from two sides; drop out the fifth portion (in the centre) and fill it up with loose earth".
- Let *OH* in figure be the radius of the circum-circle of the square *ABCD* (to be converted in to an equal circle). Leaving out one-fifth of the height (*utsedha*) *OH*, the circle drawn with the remaining height *OK* (= 4 *OH*/5) will be the required circle having the area of the given square.

Let s and r be the side of given square and radius of equal circle constructed respectively. Now,

$r = OK = (4/5)\,OH = (4/5)\,OD = (4/5)\,(s/2)\,\sqrt{2}$

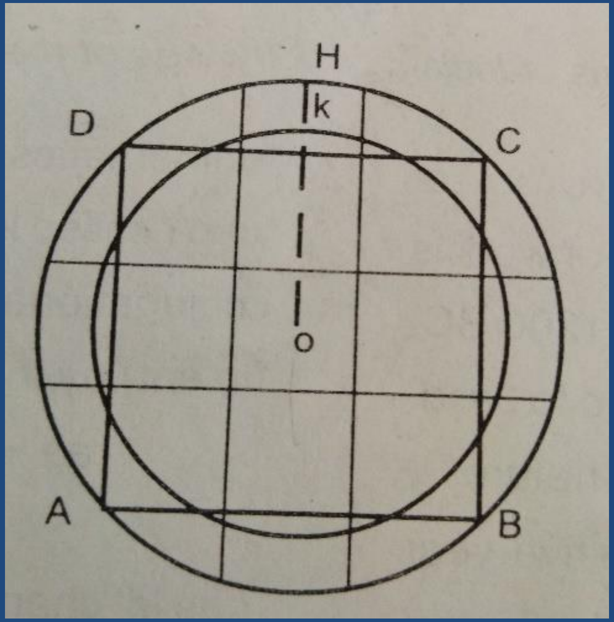
$= 2\sqrt{2}\,s/5.$

$\pi r^2 = (22/7)(2\sqrt{2}\,s/5)^2 = (176/175)\,s^2 \sim s^2.$

By construction, $\pi r^2 = s^2$.

Hence the implied approximation is found to be $\pi(2\sqrt{2}\,s/5)^2 = s^2$

i.e. $\pi = 25/8 = 3.125.$



More constructions in *Śulbasūtra*

- **Dividing a line**, circle, triangle into number of equal areas
- **Drawing a line at right angles** to a given line from a point on it and outside it;
- **Constructing a square, rectangle,trapezium, parallelogram**;
- **Constructing a square** equivalent to i) given triangles, ii) two given pentagons, iii) given rectangle; iv) a rhombus, v) isosceles triangle
- **Constructing a rectangle** equivalent to a trapezium;
- **Constructing a rhombus** equivalent to a square or rectangle and vice versa;
- **Constructing a triangle** equivalent to a square
- **Constructing a Circle** equivalent to a square & viceversa
- **Vedis** in the form of kūrma, śyena citi and so on.

THEOREM OF THE DIAGONAL OF THE RECTANGLE

- *Boudhāyana- Śulba Sūtra* (I.48)(earlier than 800 BC) gives the result, known today as Pythagoras Theorem (6th century BC):
- दीर्घचतुरश्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं करोति ।
- ' The diagonal of a rectangle produces both the areas which are produced separately by its length and breadth.'
- This theorem has to be called as *bhuja-koṭi karṇa nyāya* or as *Śulba* theorem instead of Pythagoras theorem.
- Most of the results and properties regarding the two and three dimensional figures were known to the *Śulba* Priests.

- The Jaina text (3rd cent.BCE) *Jyotiṣakaraṇḍa* which purports to expound the knowledge contained in the *Sūryaprajñapti* gives the following formulae:
- where *c* is the chord, *a* the arc, *h* the height of the segment and *d* the diameter of the circle.

$$c = \sqrt{4h(d-h)} \qquad h = \sqrt{\frac{a^2 - c^2}{6}}$$

- $$a = \sqrt{6h^2 + c^2} \qquad d = \frac{\left(c^2/_4\right) + h^2}{h}$$

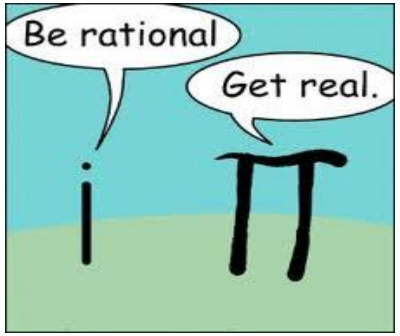
- Circumference of a circle = $\sqrt{10d^2}$
- Area of the circle = circumference × *d*/4

ĀRYABHAṬĪYA-499 CE

- The geometric concepts given in *Gaṇitapāda* of this work:
- Area of plane figures such as triangle, trapezium and circle,
- Volume(approximate) of right pyramid and sphere,
- circumference and chord of circle,
- R-sine table, shadows,
- theorems on square of hypotenuse and on square of half-chord.

CIRCUMFERENCE- DIAMETER RATIO

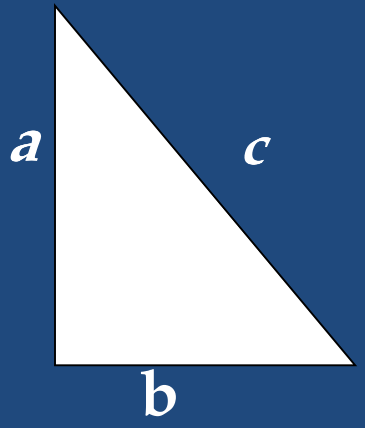
- The rule for circumference- diameter ratio is given (Ā.II.10)
- चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम् ।
  अयुतद्वयविष्कम्भस्य आसन्नो वृत्तपरिणाहः ॥
- '100 plus 4, multiplied by 8 and added to 60000; this is the nearly approximate measure of the circumference of a circle whose diameter is 20000.'
- i.e., $$\frac{circumference}{diameter} = \frac{62832}{20000} = 3.1416 = \pi \text{ value}$$



- This value first occurs in *Āryabhaṭīya*. It is noteworthy that Āryabhaṭa has specified the above value as approximate (*āsanna*).

ŚULBA THEOREM

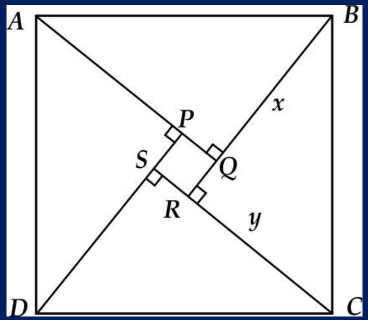
- **Bhāskara (1150 A.D.) states (L.136):**
- **तत्कृत्योर्योगपदं कर्णो दोःकर्णवर्गयोर्विवरात् ।**
  मूलं कोटिः कोटिश्रुतिकृत्योरन्तरात्पदं बाहुः ॥
- (i) $a^2 + b^2 = c^2$
  (ii) $c^2 - b^2 = a^2$
- (iii) $c^2 - a^2 = b^2$



Proof of the theorem using Areas

- Let the given triangle have base (*bhuja*) as x and altitude (*koṭi*) as y and $x>y$. By taking four triangles, which are congruent to this triangle, after juxtaposing them a quadrilateral with hypotenuses of triangles taken, as equal sides (square, as each of its angle is a right angle) is

  obtained as in Fig :

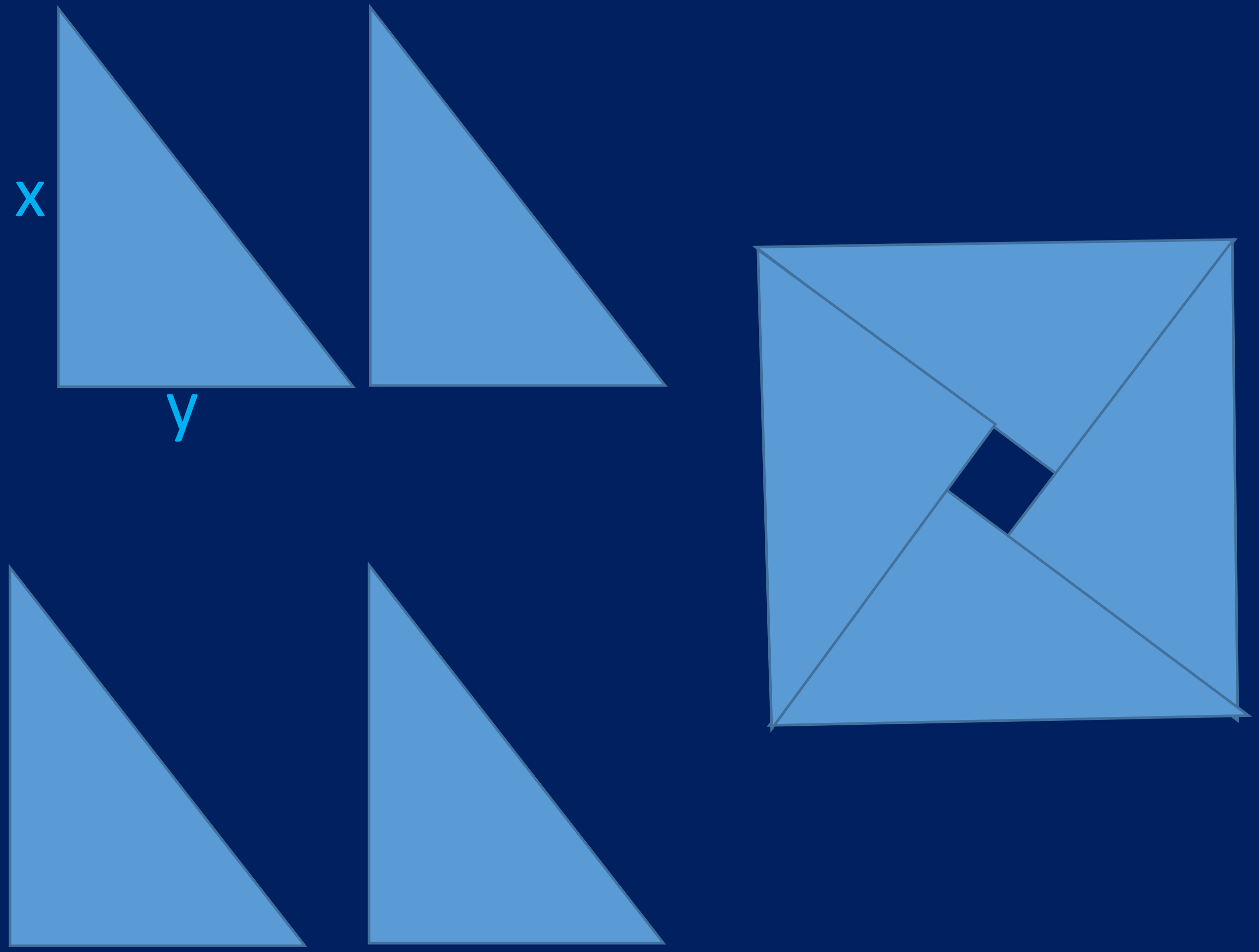


- The side of the inside square *PQRS*, $QR = BR - BQ = x - y$.
- The area of this square is $(x - y)^2$.
- The area of triangle BRC is ½ × base × altitude i.e. $\frac{1}{2} \times x \times y$.
- Sum of the areas of the four equal triangles is $4\left(\frac{1}{2} \times x \times y\right)$
- Area of the square *ABCD* = sum of areas of the four equal triangles + area of *PQRS*.

$$= 4\left(\tfrac{1}{2} \times x \times y\right) + (x - y)^2$$

$$= 2xy + x^2 + y^2 - 2xy \quad \text{(by } L\ 138\text{)}$$

$$BC^2 = x^2 + y^2$$


x
y

Monkeys problem

EXAMPLE

- वृक्षाद्धस्तशतोच्छ्रयाच्छतयुगे वापीं कपिः कोऽप्यगा-
  दुत्तीर्याथ परो द्रुतं श्रुतिपथेनोड्डीय किंचिद्द्रुमात्।
  जातैवं समता तयोर्यदि गतावुड्डीनमानं किय-
  द्विद्वंश्चेत्सुपरिश्रमोऽस्ति गणिते क्षिप्रं तदाचक्ष्व मे ॥

There was a palm tree 100 C (cubits) high and there was a well at a distance of 200 C from the tree. Two monkeys were at the top of the tree. One of them came down the tree and walked to the well. The other one jumped up and then pounced on the well along the hypotenuse. If both covered equal distances, find the length of the second monkey's jump.

SOLUTION:

Jump =DC = (tree × ground) ÷ (2 × tree + ground)

$$= \frac{100 \times 200}{(2 \times 100) + 200} =$$

$= 50$ c

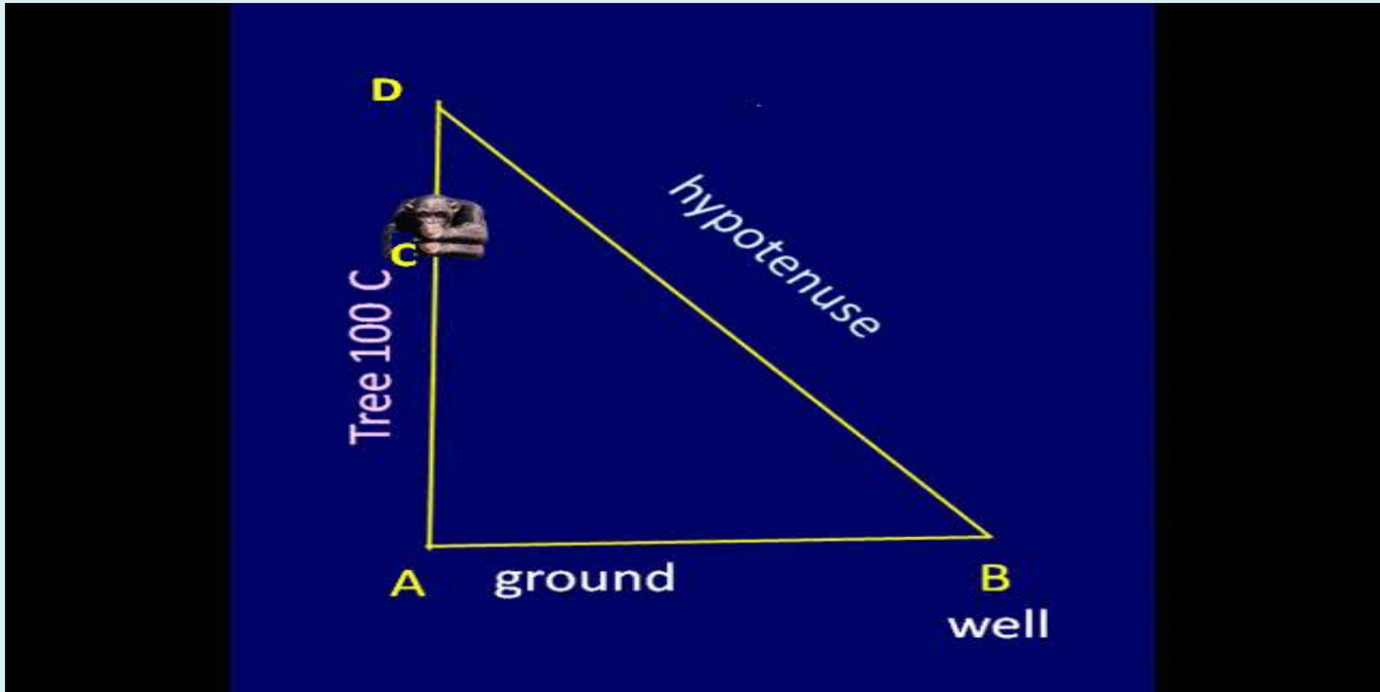
If we take the equations as

$(x + 100)^2 + 200^2 = h^2$

and $x + h = 100 + 200$

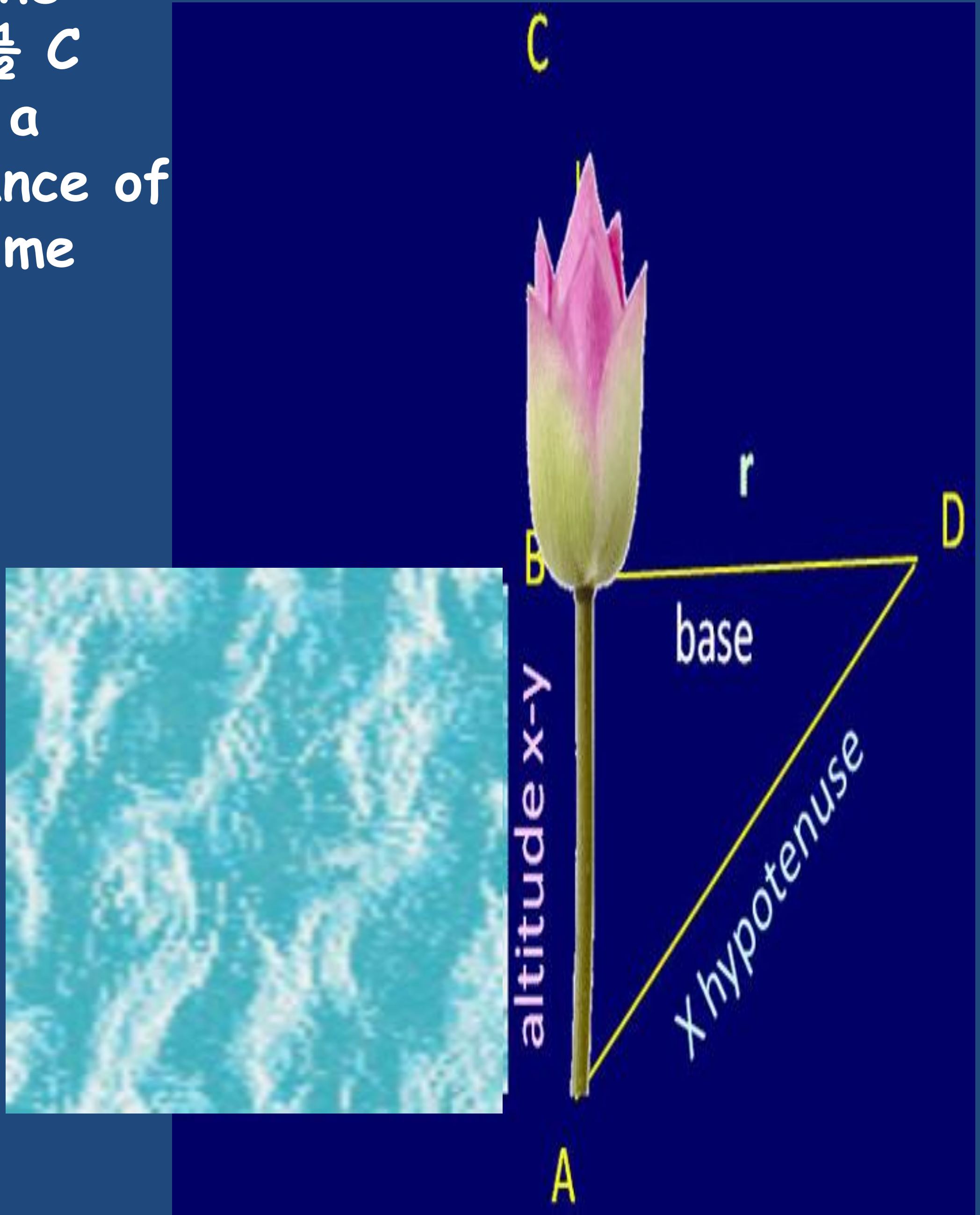
We get the same result.

An objection may be raised that the second monkey traverses a parabolic path. But as per the conditions on his movement, the path is to be taken a straight line.

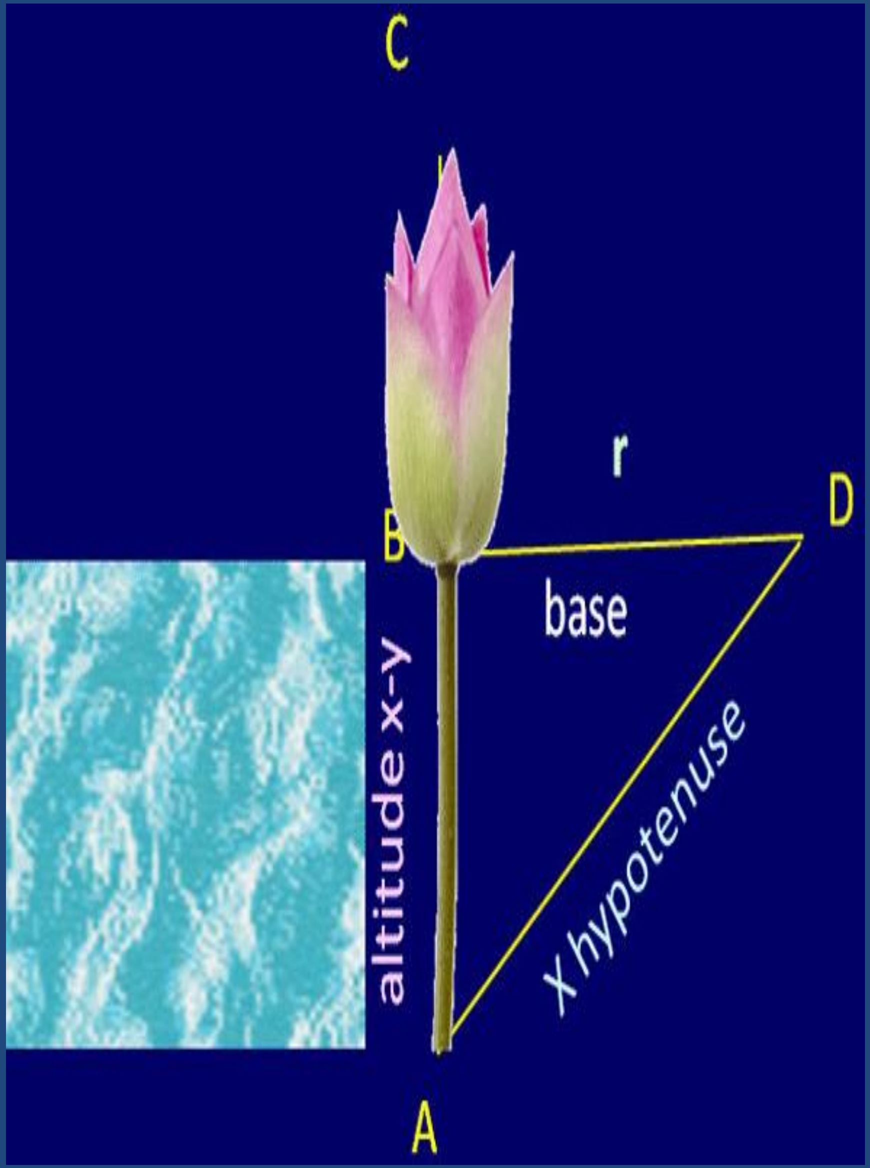


Lotus and Depth of the pond

A lotus whose height above the water surface was one vita [$\frac{1}{2}$ C (cubit)], and its tip bent by a rustling wind, sank at a distance of 2 C. O ! Mathematician, tell me quickly the depth of water.



D where the tip touches the water level, BD is called the base. The height of the lotus = the distance between B, the point where the lotus stem meets the water and C above the water surface =AD – AB = hypotenuse – altitude. Altitude (AB) is the depth of water.



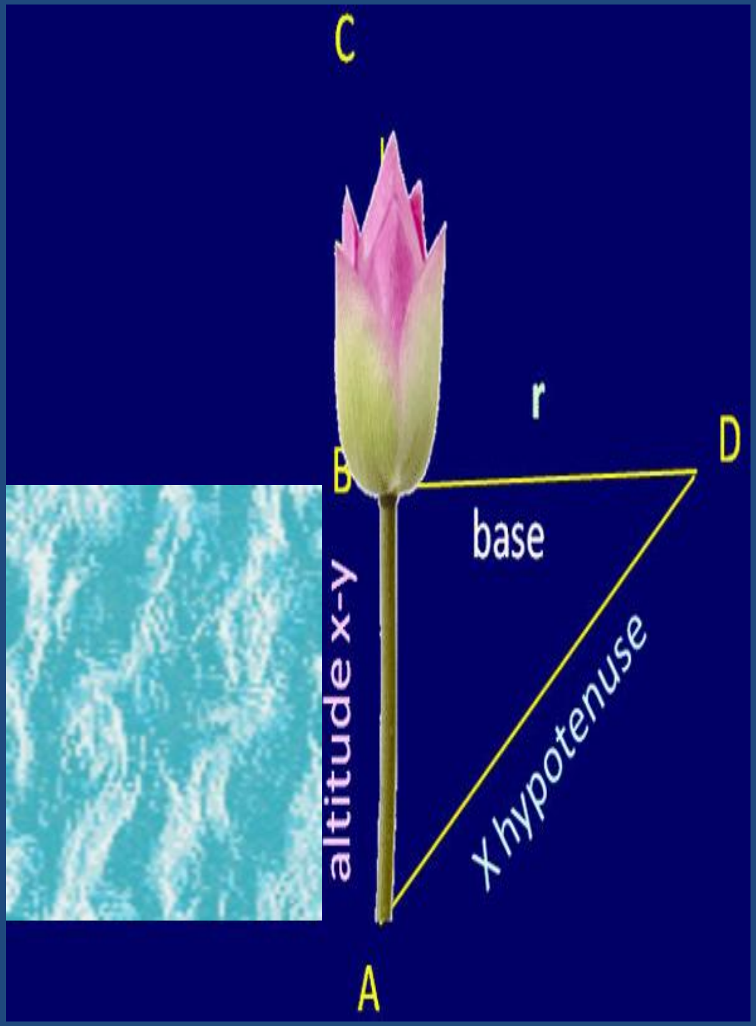
Solution: Here $(x-y)^2 + r^2 = x^2$

$\therefore x^2 - 2xy + y^2 + r^2 = x^2$

$\therefore\ x = \frac{1}{2}\ [\{(r^2) / y\} + y]$

$AB\ = x-y = [\{(r^2) / y\} + y] - y$

$AB\ = \frac{1}{2}\ [\{(r^2) / y\} - y]$



Akṣetra (Non- field)- upapatti by demonstration

- ‘Straight sticks of measures, equal to the sides of the figure, are to be arranged in the place of sides, on the ground. (i) If sum of the other sides is less than or (ii) equal to the longest side, then one end of the longest side does not touch the tip of the sum of other sides that It is very clear that it is impossible to have (closed) region within the sides, so that it is not a field(with positive area).’

- (i) $a + b < c$

- (ii) $a + b = c$

- (iii) $a + b > c$

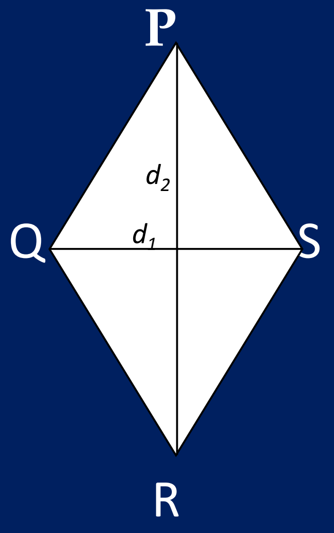
Area of Rhombus –From known result

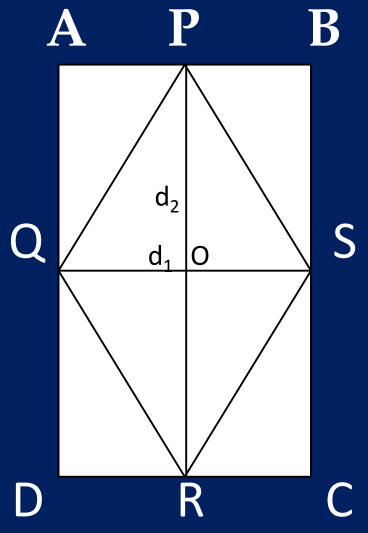
- In a rhombus, the diagonals divide it into 4 triangles. Then 4 other equal triangles are arranged by juxtaposing the sides to complete a figure ABCD (Fig) $PR \perp QS$; $\angle POQ = \angle PAQ = 90^0$ ($\Delta OQP \cong \Delta APQ$).

In the same way $\angle A = \angle B = \angle C = \angle D$. i.e. $ABCD$ is a rectangle.

Area of rectangle $ABCD$ = obtained is $d_1 \times d_2$ and is twice the area of rhombus.

Thus area of rhombus $PQRS = ½ \times d_1 \times d_2$; d_1 and d_2 being the length of the diagonals.



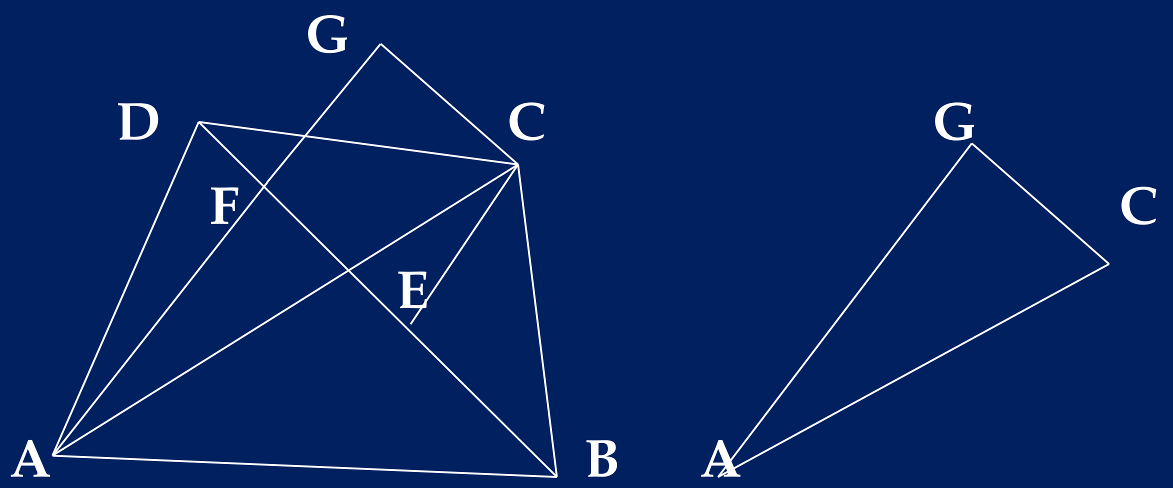


AC = AF+CE2 + EF2

Second diagonal of a quadrilateral –logical *upapatti*

- A rule is given in *L*.180- 81, when a diagonal is known, to find the second diagonal of a quadrilateral:
- In the adjoining Fig. AF &CE are perpendiculars to given diagonal BD.

To Prove : AC = $\sqrt{(AF + CE)^2 + EF^2}$



- Let $ABCD$ be a quadrilateral as in Fig..
- The diagonal BD divides the quadrilateral into two triangles. The perpendiculars drawn from A and C to the diagonal BD fall on either side of AC and meet BD at F and E respectively.
- $FG = EC$, $FG \perp FE$, $EC \perp FE$; $CEFG$ is a rectangle. So FE parallel to GC and $FE = GC$). FE, the distance between the feet of the perpendiculars is equal to altitude GC. Then $EC = FG$; $AG = AF + FG = AF + EC$. The sum of the perpendiculars, AG is the base of $\Delta\ AGC$ and AC is the hypotenuse, which is the second diagonal of $ABCD$. In right $\Delta\ AGC$,
- $AC = \sqrt{AG^2 + GC^2}$ i.e $AC = \sqrt{(AF + CE)^2 + EF^2}$

AREA

- A rule (*L*.169) to bring out the gross area of a quadrilateral and area of a triangle:
- सर्वदोर्युतिदलं चतुःस्थितं बाहुभिर्विरहितं च तद्वधात्।
  मूलमस्फुटफलं चतुर्भुजे स्पष्टमेवमुदितं त्रिबाहुके ॥
- When the sides of the quadrilateral a, b, c, d and s is the semi-perimeter, then
- Area of quadrilateral(cyclic) $=\sqrt{(s-a)(s-b)(s-c)(s-d)}$
- Area of triangle $= \sqrt{(s-a)(s-b)(s-c)(s)}$
- Brahmagupta was the first to give the above formulae and also the expression for the diagonals of a quadrilateral.(Br.Sp.Si.XII.28)

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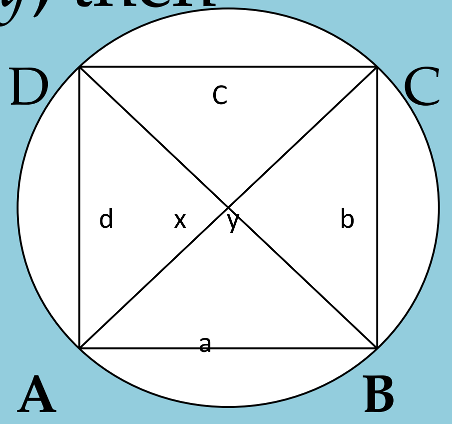
DIAGONALS OF QUADRILATERAL

- कर्णाश्रित भुजघातैक्यमुभयथान्योन्य भाजितं गुणयेत् ।
  योगेन भुजप्रतिभुजवधयोः कर्णौ पदे विषमे ॥
- The above verse gives the following result. In a cyclic quadrilateral ABCD, if a,b,c,d are the lengths of the sides AB,BC,CD and DA respectively and its diagonals AC = x and BD = y, then
- $$AC = x = \sqrt{\frac{(ad+bc)(ac+bd)}{ab+cd}}$$
- $$BD = y = \sqrt{\frac{(ab+cd)(ac+bd)}{(ad+bc)}}$$



- **M.Eves in '*An Introduction To History Of Mathematics*' (New York,1969) says about** the above result as 'most remarkable in Hindu geometry and solitary in its excellence.'
- 'The above formula was rediscovered in Europe a thousand years later by W.Snell about 1619 AD.'

DEMONSRATION AND GEOMETRC PROOF

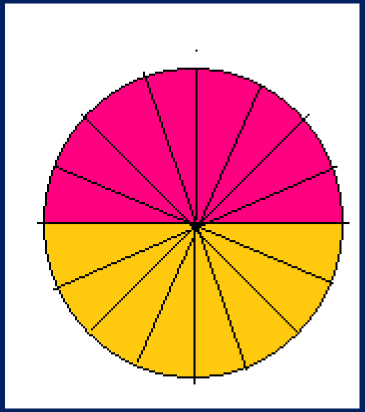
- **Bhāskara gives a rule for finding the area of a circle and finding the surface area and volume of a sphere (*L.*201)**

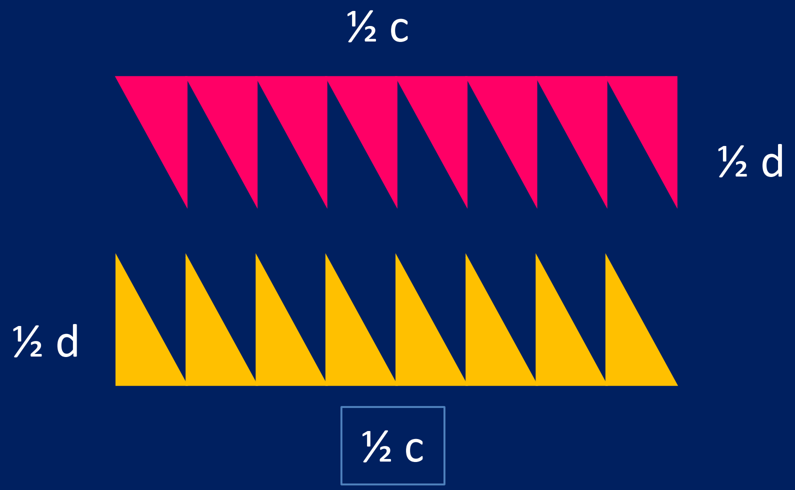
- वृत्तक्षेत्र परिधिगुणितव्यासपादः फलं यत्
  क्षुण्णं वेदैरुपरि परितः कन्दुकस्येव जालम् ।
  गोलस्यैवं तदपि च फलं पृष्ठजं व्यासनिघ्नं
  षड्भिर्भक्तं भवति नियतं गोलगर्भे घनाख्यम् ॥

- Area of circle = ¼ of diameter x circumference
- = ¼ d x c $=\pi r^2$
- Surface area of sphere = (¼ d x c) x 4 = $4\pi r^2$
- Volume of sphere = (¼ d x c) x 4 x d/6 = $\frac{4}{3}\pi r^3$

- AREA OF A CIRCLE
- The circular field is to be divided into two equal halves.
- Divide this into conical sections, as many pieces as possible .
- Arrange them in such a way that a rectangle is formed.

DEMONSTRATION



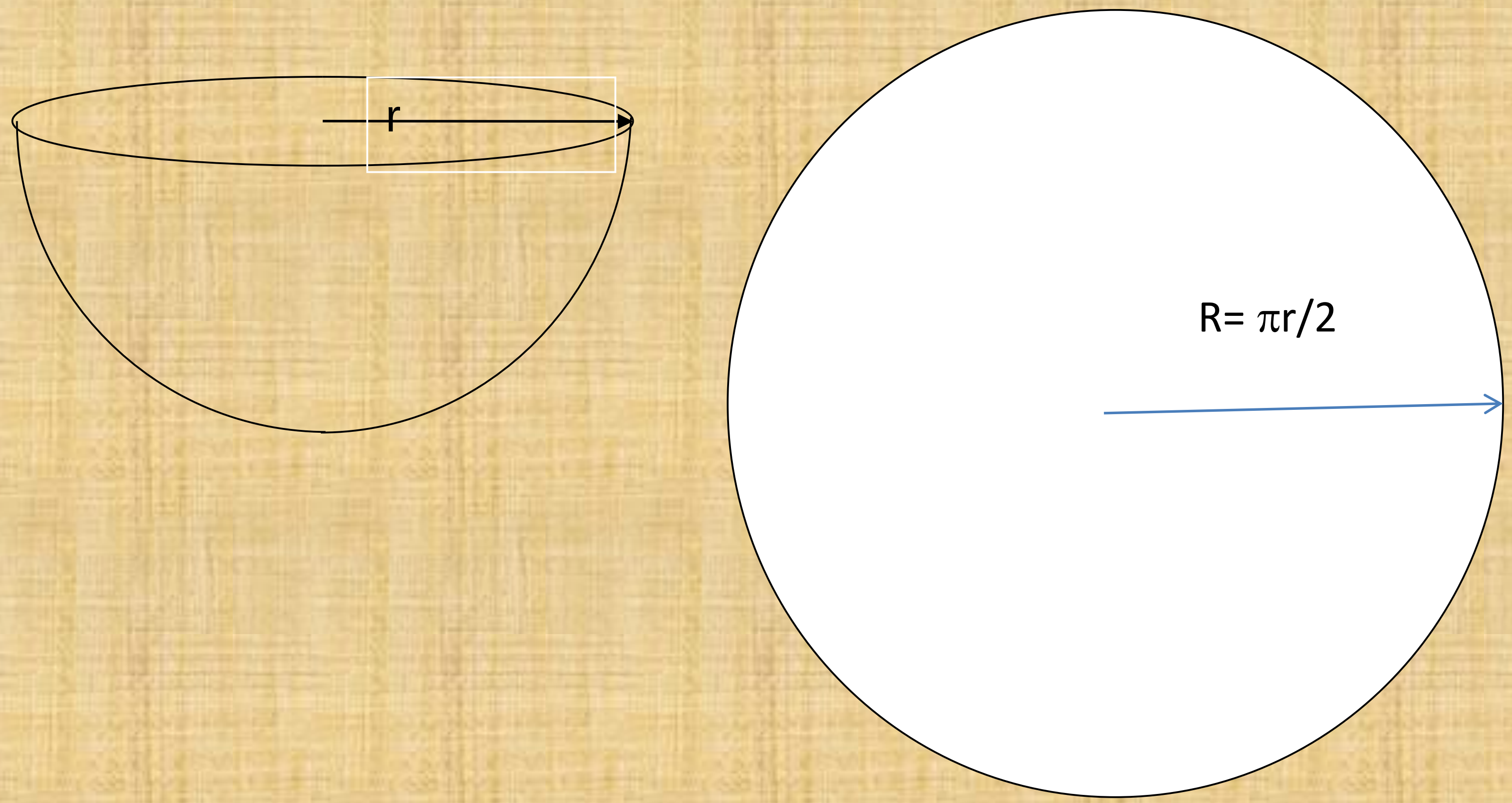


For this rectangle, one side is half of diameter and other side is half of circumference.

The area of circle = area of rectangle formed

$$= \frac{1}{2} d \times \frac{1}{2} c = \frac{1}{4}\, 2r \times 2\pi r = \pi r^2$$

- SURFACE AREA OF A SPHERE
- A circular cloth whose diameter is equal to half of the circumference of the greater circle of the sphere is taken.
- Half the sphere is covered by this circular cloth.
- After covering the hemisphere, a small piece of cloth just like a waist-belt is left.
- This extra cloth subtracted from the circular cloth is the surface area of hemisphere.


r
R= πr/2

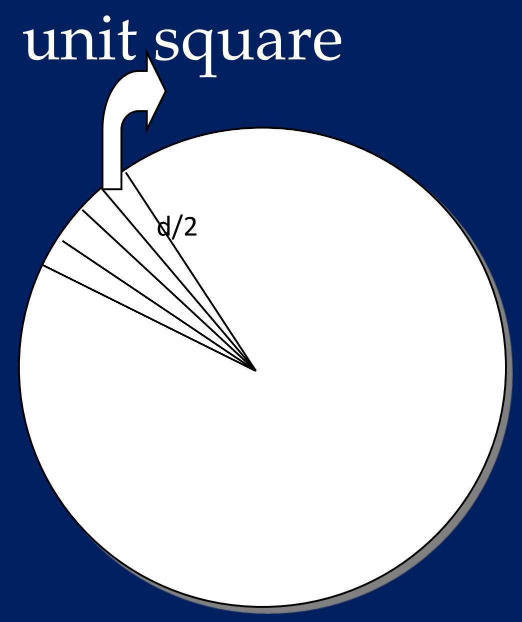
- Area of circular cloth = $\frac{1}{4}$ d x C = $\frac{1}{4}$ d x πd
- = $\frac{1}{4}$ x $\frac{1}{2}$ c x π x $\frac{1}{2}$ x C
- = $\frac{1}{4}$ π x $\pi^2 r^2$
- ~ (5/2) πr^2 as π^2 ~ 10

- The area of extra remaining cloth = $\frac{1}{2}$ the area of base circle = $\frac{1}{2}$ πr^2
- Area of hemisphere = (5/2) πr^2 --$\frac{1}{2}$ πr^2= 2 πr^2
- $\Rightarrow$ Area of a sphere = 4 πr^2

- VOLUME OF A SPHERE
- The whole surface area of the sphere is divided into unit squares.
- Thus there are $4\pi r^2$ unit squares.
- Corresponding to each unit square, one thin conical section is made.
- The base surface of each cone is a square of unit length i.e. its area is one unit square.
- This has depth equal to half of the diameter.

- Volume of 1 conical solid = base area x depth x 1/3
- Volume of 1 conical section made

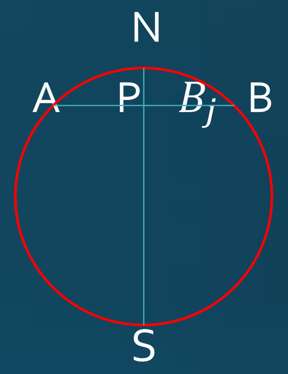
  = 1 unit square x (d/2) x (1/3) = d/6
- Volume of 4 πr^2 conical sections = 4 πr^2 x (d/6)
- = 4 πr^2 x (2r/6)
- = (4 πr^3 /3) cubical units



Yuktibhāṣā

- *Yuktibhāṣā* of Jyeṣṭhadeva (1500 – 1610 AD) derives the expressions for the formulae for volume of a sphere etc. with the help of the methods of Calculus, which were rediscovered in Europe later by Newton and Leibnitz (17^{th}-18^{th} CE).
- The derivation of volume of a sphere as in *Gaṇita-yukti-bhāṣā* , is as follows:
- Let r be the radius of the sphere and C , the circumference of a great circle.
- Area of circle = (1/2) C × r (1)
- The half-chord B_j is the radius of the j-th slice into which the sphere has been divided. The corresponding circumference is $\left(\frac{C}{r}\right) B_j$ and from (1), the area of this circular slice is $= \frac{1}{2}\left(\frac{C}{r}\right) B_j^2$
- If Δ is the thickness of the slices, then the volume of the j-th slice is $= \frac{1}{2}\left(\frac{C}{r}\right) B_j^2\, \Delta$

- Volume of the sphere = the sum of the squares of the Rsines B_j^2
- V $\approx \frac{1}{2}\left(\frac{C}{r}\right)[B_1^2 + B_2^2 + \cdots B_n^2]\Delta$ (2)
-
-



-
-
- Fig.Square of half-chord equals product of *śaras*
- In Fig. $AP = PB = B_j$ = jth half-chord, starting from N, the north point.
- $B_j^2 = AP \times PB = NP \times SP$ (by *Āryabhaṭīya* rule Gaṇita 17)
- $= \frac{1}{2}\,[(NP + SP)^2 - (NP^2 + SP^2)]$
- $= \frac{1}{2}\,[(2R)^2 - (NP^2 + SP^2)]$ (3)
-

- If $\Delta = \frac{2r}{n}$, be the thickness of each slice, the j-th Rversine

NP = $j\,\Delta$ and its complement SP = $(n-j)\,\Delta$. Hence , while summing the squares of the Rsines B_j^2, both NP^2 and SP^2 add to the same result. Thus by (2) and (3)

- $V \approx \frac{1}{2}\left(\frac{C}{r}\right)\left(\frac{2r}{n}\right)\left(\frac{1}{2}\right)\left[(2r)^2 + (2r)^2 \ldots + (2r)^2\right]$

$$-\frac{1}{2}\left(\frac{C}{r}\right)\left(\frac{2r}{n}\right)\left(\frac{1}{2}\right)\left(\frac{2r}{n}\right)(2)\left[(1)^2 + (2)^2 \ldots + (n)^2\right] \qquad (4)$$

- For large n, the sum of the squares (*varga-saṅkalita*) is essentially one-third the cube of the number of terms. Then (4) becomes

- $V = \left(\frac{C}{2r}\right)\left(4r^3 - \frac{8r^3}{3}\right)$
- $= \left(\frac{C}{6}\right) d^2 = \frac{2\pi r}{6} \times (2r)^2 = \frac{4}{3}\,\pi\, r^3$
- Hence Volume of sphere = one-sixth of circumference × square of diameter, which is same as V = $\frac{4}{3}\,\pi\, r^3$

- Moreover there are rules, examples and demonstrations to find
- the arrow,
- chord and diameter of a circle,
- sides of regular polygon inscribed in a circle,
- arc length from chord,
- area of segment etc.

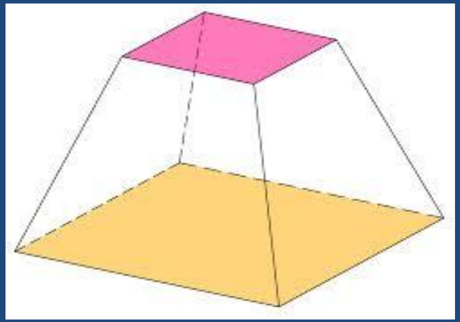
KHĀTA VYAVAHĀRA

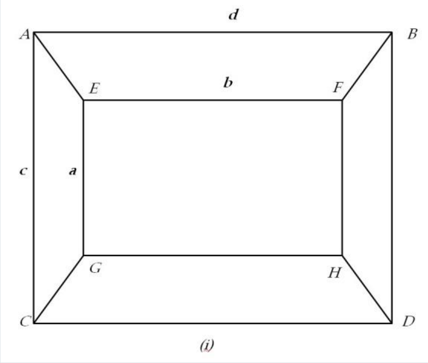
- This chapter he deals with excavations (*khāta*), stacks of brick and the like, sawing of timber and stores of grain.
- Rule (L.214) for volume of irregular solid :
- गणयित्वा विस्तारं बहुषु स्थानेषु तद्युतिर्भाज्या ।
  स्थानकमित्या सममितिरेवं दैर्घ्ये च वेधे च ॥
  क्षेत्रफलं वेधगुणं खाते घनहस्तसंख्या स्यात् ॥

$$l = \frac{l_1 + l_2 + \cdots + l_q}{q} \qquad b = \frac{b_1 + b_2 + \cdots + b_p}{p} \qquad h = \frac{h_1 + h_2 + \cdots + h_r}{r}$$

- Volume = *lbh*

- To find the volume of a pyramid and its frustum, a rule is given (*L*.217) :
- मुखजतलजतद्युतिजक्षेत्रफलैक्यं हृतं षड्भिः ।
  क्षेत्रफलं सममेवं वेधहतं घनफलं स्पष्टम् ॥
  समखातफलत्र्यंशः सूचीखाते फलं भवति ॥
- Volume (V) of a frustum with similar rectangular faces of sides, 'a,b' and 'c,d'; depth 'h' is given by
- $V = \{ ab + cd + (a + c)(b + d)\} (h/6)$





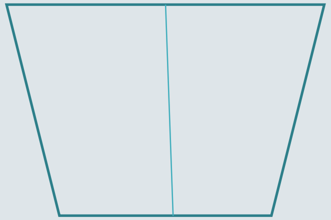
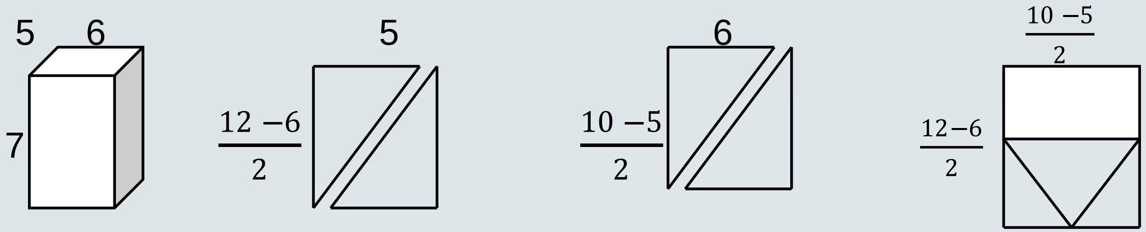
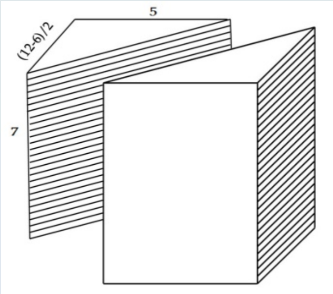


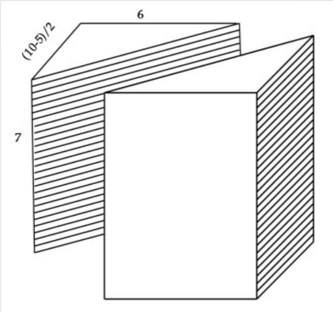
Fig. 10.

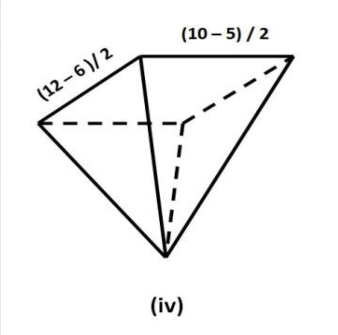


(i) (ii) (iii) (iv)

Fig.10a (ii) (iii) (iv)







- A frustum is with bottom base as $a \times b = 5 \times 6$ and top face $c \times d = 10 \times 12$ and depth 7 units.
- *BV* says (a) A rectangular cuboid (samakhāta) of sides $a = 5$ and $b = 6$ and depth $h = 7$ units inserted (b) Four triangular prisms(*sūcī-khātas*) on the 4 vertical face. (c) Four rectangular pyramids at the 4 corners of the top face of the box.
- In the middle, there is a samakhāta with sides 5 , 6 and depth 7. [Fig. (i)]. Its volume = $5 \times 6 \times 7 = 210$.
- 2 *sūcī-khātas* on the side 5, with volume $2 \times \{7 \times \frac{5}{2} \times \frac{12-6}{2}\} =$

  $2 \times \frac{105}{2} = 105$[Fig. (ii)]
- 2 *sūcī khātas* on the side 6, with volume = $2 \times \{ 7 \times \frac{6}{2} \times \frac{10-5}{2}\} = 2 \times \frac{105}{2} = 105$ [shown in Fig. (iii)]. Two such sections together make a parallelopiped; so each is half the volume of a parallelopiped.
- 4 rectangular pyramids with volume $= 4 \times \{ \frac{7}{3} \times \frac{12-6}{2} \times \frac{10-5}{2}\} = 4 \times \frac{35}{2} = 70$ [Fig (iv)]. Three such sections together make a parallelopiped; so each is one third of the volume.
- The sum of the volumes is the volume of the frustum

$= 210 + 105 + 105 + 70 = 490$ cu.units

- volume of rectangular box, $V_1 = abh$
- Volume of 4 triangular prisms, $V_2 = 2 \cdot \frac{a}{2} \cdot \frac{d-b}{2} \cdot h + 2 \cdot \frac{b}{2} \cdot \frac{c-a}{2} \cdot h.$
- Volume of 4 rectangular pyramids, $V_3 = 4 \cdot \frac{c-a}{2} \cdot \frac{d-b}{2} \cdot \frac{h}{3}$

- Adding the 3 volumes
- $V = V_1 + V_2 + V_3 = abh + 2 \cdot \frac{a}{2} \cdot \frac{d-b}{2} \cdot h + 2 \cdot \frac{b}{2} \cdot \frac{c-a}{2} \cdot h + 4 \cdot \frac{c-a}{2} \cdot \frac{d-b}{2} \cdot \frac{h}{3}$
- $= h \left(\frac{ad}{6} + \frac{bc}{6} + \frac{cd}{3} + \frac{ab}{3}\right)$
- $= \frac{h}{6}(ab + cd + ad + ab + bc + cd)$
- $= \frac{h}{6}\{ab + cd + (a+c)(b+d)\}$
- Volume required in the example (*L.* 218) is
- $\frac{7}{6}\{5 \times 6 + 10 \times 12 + (5+10)(6+12)\} = \frac{1}{6} \times 2940 = 490.$

- Figures, as explained in *Buddhivilāsinī* , give the vertical cross section of the excavation as looked from top and the horizontal cross section. In modern Engineering drawing these are said to be 'Plan and Elevation.'

EXAMPLE

- There is a well in the shape of a frustum of a pyramid. Its top is a rectangle of sides 10 and 12 cubits. The base is half the size of the top i.e. 5 and 6 cubits. If its height is 7 cubit, O friend! Find the volume of the well.
- Solution: Volume $= \{ ab + cd + (a + c)(b + d)\} (h/6)$

$$= \{ 10\times 12 + 5\times 6 + (10 + 5)(12 + 6)\} (7/6)$$

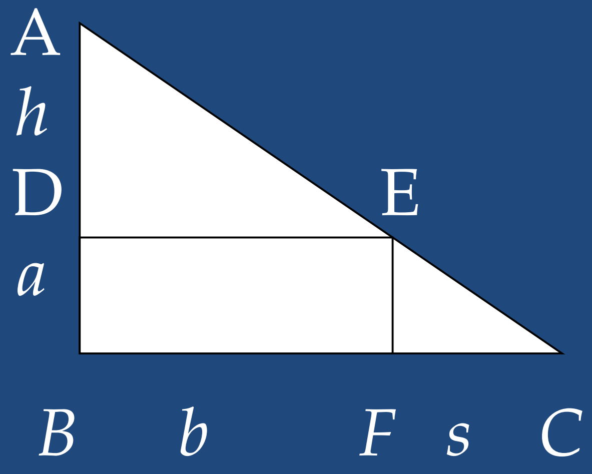
$$= 490 \text{ cubic cubits.}$$

- *Gaṇita Sāra Saṅgraha* (*GSS*) (850 AD) gives a rule for finding the volume of frustum-like solid in 3½ verses (GSS.VIII.9-11½)

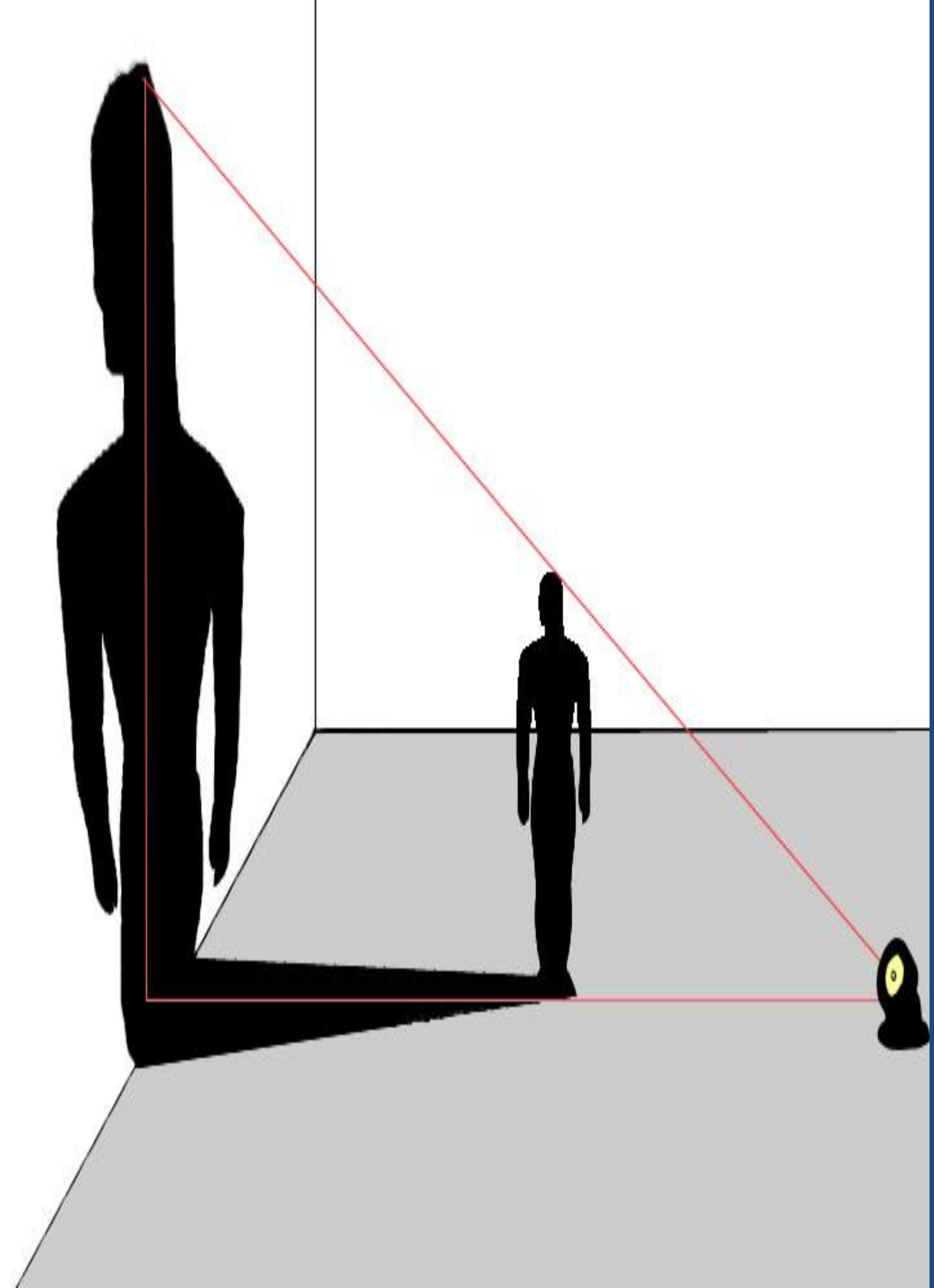
SHADOWS AND GNOMON

- A rule (L.234) to find the length of shadow when the distance between the lamp-post and gnomon is given:
- शङ्कुः प्रदीपतलशङ्कुतलान्तरघ्नः ।
  छाया भवेद्विनरदीपशिखौच्च्यभक्तः ॥२३४ ॥

- Distance (b) between the foot of the lamp-post and foot of the pole is multiplied by pole-height (a), divided by the difference in height (h) of the lamp-post and pole to get the shadow.



- Shadow (s) = $(a \times b)/h$
- $(\Delta\, EFC \sim \Delta\, ADE \Rightarrow s/b = a/h)$

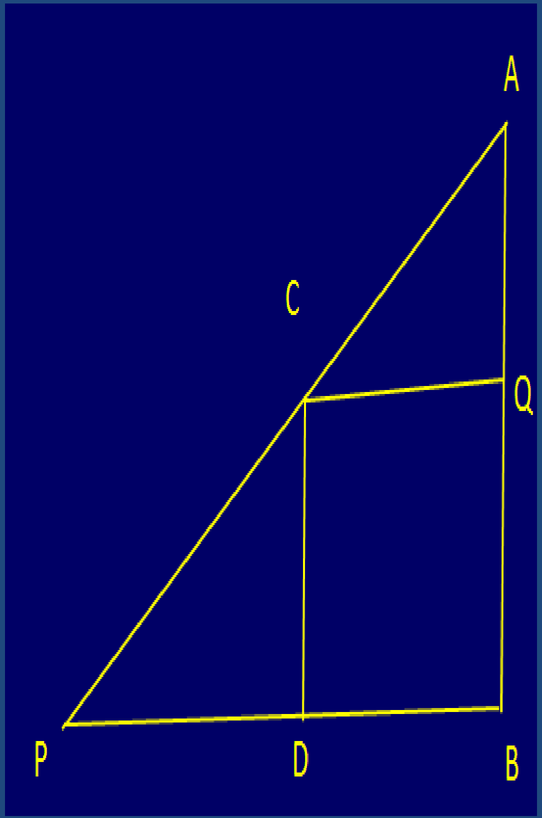


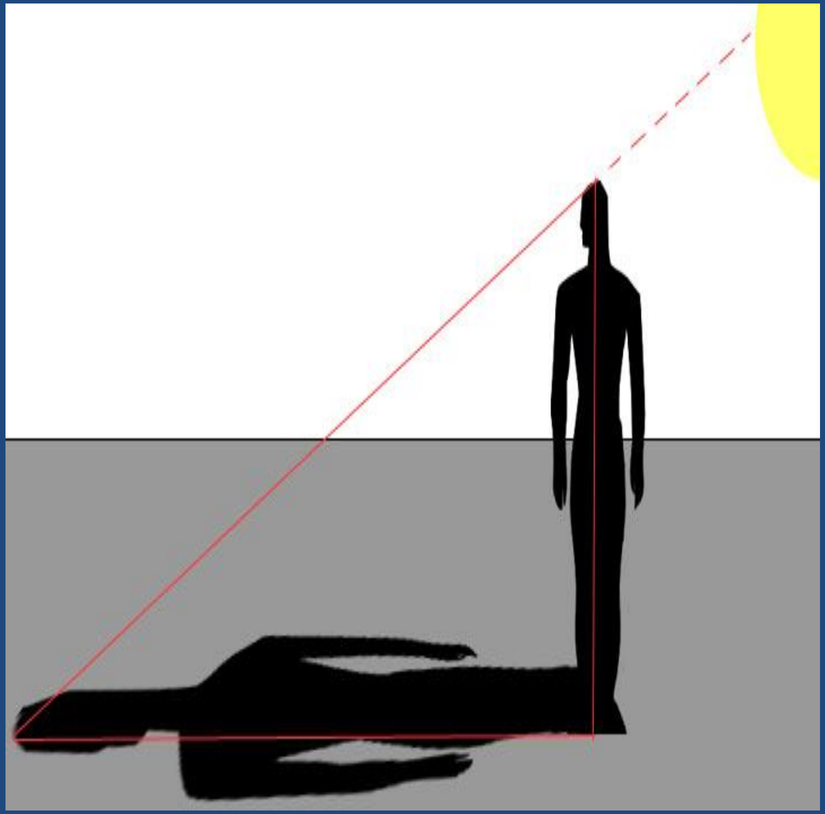
Shadow problem

If BD = 3 C [cubits = 72 angulas], AB = 3½ C (84A) and CD = 12A. Find the length of the shadow.

Solution: $PD = \dfrac{CD \times CQ}{AQ}$ where CQ =BD =72 and AQ =84 - 12

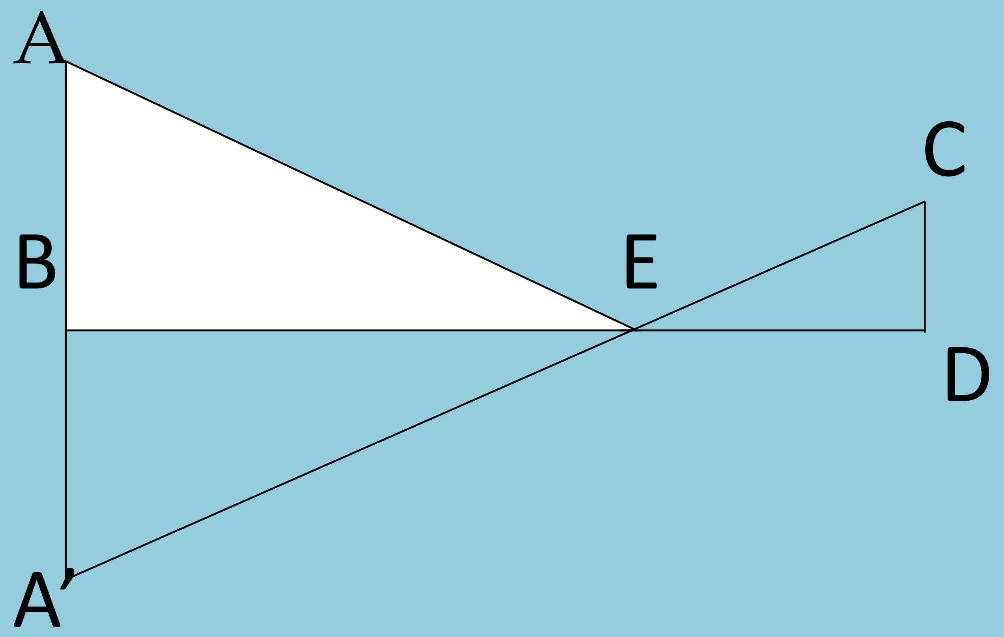
$= \dfrac{12 \times 72}{84 - 12} = 12\ A$





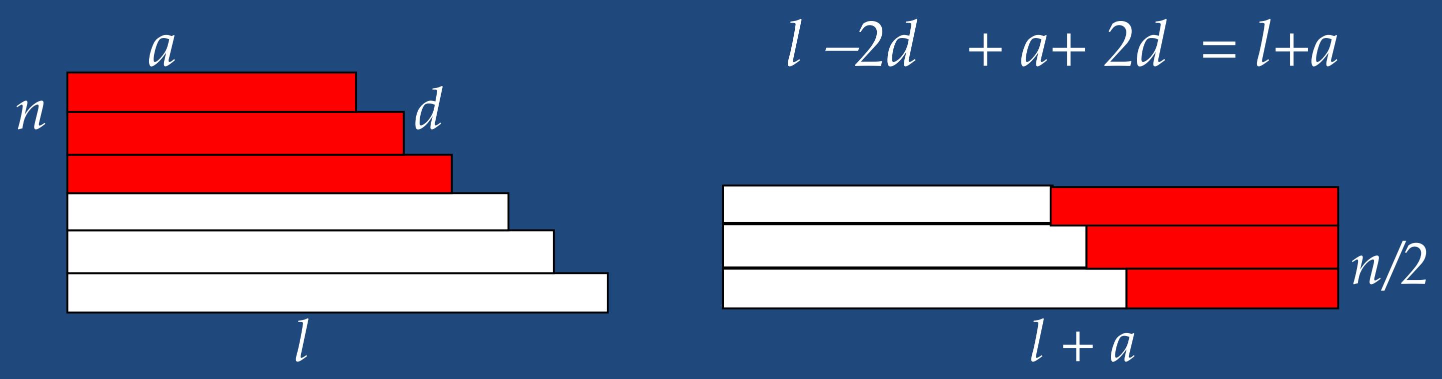
- *Brāhma- Sphuṭa- Siddhānta* gives varieties of problems on shadows.
- To find the distance of reflecting water from the house (*BSS*.XIX.17):
- युतदृष्टिगृहौच्चयहृता ह्यन्तरभोमिर्दृगौच्चयसङ्गुणिता ।
  फलभूर्न्यस्ते तोये प्रतिख्याग्रं गृहस्य नरात् ॥
- The distance between the house (AB) and the man is divided by the sum of the heights of the house and the man's eyes, multiplied by the height of the eyes (CD). The tip of the reflection of the house will be seen when the reflecting water is at a distance equal to the above product.
- If (E) is the reflecting point, the man will be able to see the tip of the image (A′), when DE = $\frac{BD \times CD}{AB + CD}$

- $\Delta ABE \sim \Delta CDE \Rightarrow CD/AB = DE/BE$
- $\Rightarrow CD/(AB + CD) = DE/(BE + DE) = DE/BD$
- $\Rightarrow DE = \dfrac{BD \times CD}{AB + CD}$
- Also, $AB/CD = BE/DE \Rightarrow$ Height of the house = $AB = (BE \times CD)/DE$



GEOMETRICAL ALGEBRA

- Nīlakaṇṭha in his *Āryabahṭīyabhāṣya* (p.72) introduces *Śreḍhīkṣetra* to establish the summation formula of Arithmetic progression.



- Area= $n/2\ (l+a)$ = sum of the series
- *Pāṭīgaṇitam* (rules 80- 82) also describes the construction of *Śreḍhīkṣetras* (series figures)

Foreign Scholars of Indian Mathematics and Astronomy

- Some wellknown professors:
- H. Colebrooke – United kingdom(19^{th} cent.)
- David Pingre – United States (20^{th} cent.)
- Takao Hayashi – Japan
- Michio Yano – Japan
- Kusuba – Japan
- Kim Plofker – United States
- Clemency – New Zealand
- When foreign scholars are learning Sanskrit and building up their career on Indian mathematics and astronomy why not Indians?

What is needed

- In TheTimes of India (Kolkata, August17,p10), there appeared an interview of Manjul Bhargava, who was awarded the Fields Medal in 2014.
- When asked: Why is India still a middlepower in mathematics despite its famed legacy?
- The reply that Manjul gave was:
- Students in India should be taught about the great ancient Indian mathematicians like Panini, Pingala, Hemachandra, Aryabhata and Bhaskara. Their stories and works inspired me, and I think they would inspire students across India. Many of these works were written in Indian languages in beautiful poetry, and contain important breakthroughs in the history of mathematics.

- Most of the Indians do not know that a vast ancient scientific literature including books on mathematics, science etc. exist in India.
- The purpose of the workshop :
- to make the youngsters know about the greatness of mathematicians and mathematics that existed in India centuries ago, while the other country men did not even know how to calculate,
- to boost up their self-confidence when they think that if their ancestors could excel the rest of the world, they also can and
- to initiate them into new ideas and research.

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THANK YOU